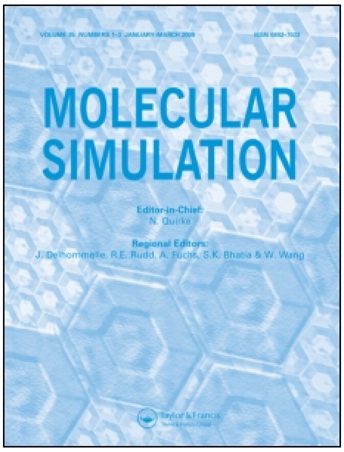

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Statistical properties of granular gas under microgravity one-dimensional inelastic hard rod system

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Statistical properties of granular gas under microgravity one-dimensional inelastic hard rod system

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We have studied numerically statistical properties of granular gas in a one-dimensional inelastic (viscoelastic) hard rod model under microgravity, which is designed to the mimic experimental granular-vibrated beds by introducing a velocity-dependent restitution coefficient (VDRC). Our systematic simulations show that various macroscopic properties of this model are quantitatively different from a linear combination of the previous simulations based on the constant restitution coefficient (CRC). The present results are significantly important to study a vibration response and dynamics of granular gas especially in microgravity experiment.

Keywords: Microgravity; Granular gas; Event-driven molecular dynamics simulation; Velocity-dependent restitution coefficient; Granular-vibrated beds; Non-equilibrium steady state

1. Introduction

Recently, the granular gas dynamics have been actively studied as a prototype for the development of local nonequilibrium statistical physics [1]. It has been known that the differences of boundary and external field cause a significant influence on the behaviour of granular gas. Since there are big possibilities to create new industry in microgravity environment by applying the fluidisation and the pattern formation of the granular-vibrated beds, the systematic study of granular gas dynamics under microgravity is an important issue from the viewpoint of transport engineering. It is necessary to investigate the vibration response of granular gas under microgravity and to explore the suitable control parameters in the nonequiribrium steady state (NESS).

The many important studies of the granular-vibrated beds have been published in the past decade. By the presence of gravity g, it is easy to perform an experiment in NESS, in which energy is injected by a vibrating plate and is dissipated through inelastic collisions between particles. The non-dimensional control parameter to characterize the transition between condensed and fluidised states is known as the maximum accelerations divided by the gravity $\Gamma = A_0\omega^2/g$, where A_0 and ω are the amplitude and the angular frequency of the vibrating plate, respectively. Under gravity on the ground, it has been shown that the periodical various patterns appear in the thin granular layer driven by vertical vibration, in which the phase diagram of those patterns in two parameter space (Γ, ω) were studied in detail [2]. Recently, in the experiment of quasi-two-dimensional granular layer, it was also shown that the ripples and the undulations pattern appear, which are independent of the separation distance of the container [3].

In a one-dimensional inelastic hard rod system (the simplest granular-vibrated beds), scaling behaviours of macroscopic properties (such as the centre of mass) were shown by Luding *et al.* [4], in which the various properties are scaled by functions $F(N, r_c)$ and $A_0\omega$, where N and r_c are the particle number (layer) and the constant restitution coefficient (CRC), respectively. These facts showed that if (N, r_c) and Γ are fixed, the macroscopic properties are determined as a certain fixed value for any (g, ω, A_0). Actually, many previous studies (including the works of authors in microgravity [5,6]) have been performed by using CRC for its simplicity. However, in experimental and theoretical works, it is generally known that the restitution coefficient strongly depends on the impact velocity v_{imp} (e.g. [7–10]). Hence, in case of the velocity-dependent restitution coefficient (VDRC) $r(v_{imp})$, the influence to the dynamics and macroscopic properties

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of granular gas under the competition of two time scales between VDRC $r(v_{\rm imp})$ and gravity g have not been understood yet. McNamara and Falcon [11] introduced VDRC instead of CRC in their simulations of the strongly excited (i.e. high velocity impact) two-dimensional vibrated beds in a confined box with and without gravity. They investigated highly excited state with a maximum velocity of the vibrated plate $A_0\omega = 1.6 \sim 3.1$ with VDRC and obtained different results from that of CRC. On the other hand, since time scale becomes large under microgravity, the low frequency excitation (i.e. low impact velocity) becomes dominant in VDRC and the relationship between VDRC (collision rule) and the macroscopic properties is not clear.

In this paper, we performed an event-driven molecular dynamics simulation (EDMD) [12] systematically to investigate granular gas dynamics under microgravity in the simple one-dimensional inelastic (viscoelastic) hard rod model, which is designed to mimic the experimental granular-vibrated beds by introducing VDRC.

2. Model & numerical settings

The schematic figure of the present study is shown in figure 1, in which the plate at the bottom is vibrated sinusoidally with $z_0(t) = A_0 \sin \omega t$, where A_0, ω, t are amplitude, angular frequency and time, respectively. This model is composed of N inelastic hard rod in a line with a diameter d under the gravity g.

To compare various quantitative properties obtained from our work with the previous ones [4], we fixed parameters $(N, A_0) = (10, d)$ and changed (ω, g) throughout the paper.

When the particles collide inelastically, the restitution coefficient r is defined from the relative velocity before and after the collision. In many previous studies, the restitution coefficient $r(< 1)$ is a constant value and is independent of the impact velocity. However, many impact experiments show that the restitution coefficient

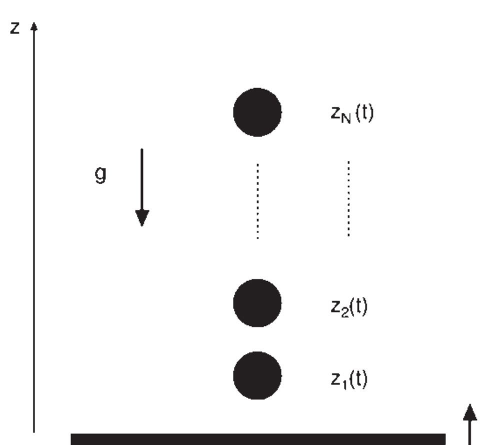

Figure 1. One-dimensional inelastic hard rod model under gravity (the simplest granular-vibrated beds).

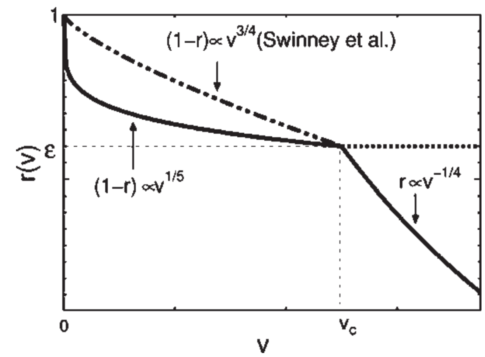

Figure 2. The VDRC $r(v)$ as a function of relative impact velocity v between particles (see equation (1)) is shown. The solid line is obtained from the impact experiments for steel particles.

of granular material strongly depend on its impact velocity. We investigated the present model with VDRC by taking into account both the viscoelastic and plastic deformations of particles, which correspond to low and high impact velocities, respectively.

In the experiments for steel spheres, VDRC $r(v)$ as a function of relative impact velocity v between particles are well described by $r(v) \propto v^{-1/4}$ for plastic deformations and $(1 - r) \propto v^{1/5}$ for viscoleastic dissipation of elastic deformation [11] (figure 2). Two functions of VDRC are connected at the boundaries (v_c, ε), which can be regarded as the material constants of steel spheres.

In microgravity, since the viscoelastic dissipation of elastic deformation might be dominant, we do not consider the plastic deformation. Thus, we used the following functions of VDRC for simplicity.

$$r(v) = \begin{cases} 1 - (1-\varepsilon)\left(\frac{v}{v_c}\right)^{1/5} & (v < v_c) \\ \varepsilon & (v \geq v_c) \end{cases} \tag{1}$$

In $v \geq v_c$, we used CRC at the value of $\varepsilon = 0.92$. This is because of the restitution coefficient of the steel spheres under the gravity on a ground being $\varepsilon \simeq 0.90$ in the actual experiments. Moreover, it is convenient for comparison that this value is the same as the previous studies of CRC by Luding *et al.* ($r_c = 0.92$) [4].

The other material constant v_c can be defined by $v_c = \sqrt{gd}$. In the gravity on the ground $g = 9.8\,\mathrm{m/s^2}$ and the diameter $d = 0.01$ m, we obtain $v_c \simeq 0.3$ m/s which is the almost same value of the steel sphere experiment described in the paper of McNamara and Falcon [11]. Here, we consider the physical meanings of the order v_c. In case of $\Gamma \sim 1$ (i.e. $g \sim A_0\omega^2$) under the assumption of $A_0 \sim d$, we obtain $v_c \sim A_0\omega$. Thus, the material constant v_c becomes same order as the maximum velocity of vibrated plate $v_{\max} = A_0\omega$. Therefore, if the material constant v_c is fixed for all simulations, we can discuss the numerical results of granular-vibrated beds for various parameters based on the reference state with gravity g and $\Gamma \sim 1$.

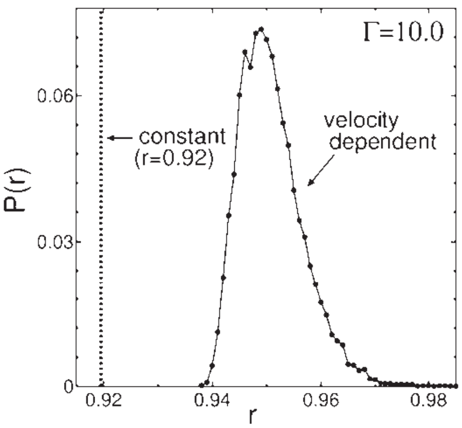

Figure 3. PDF of restitution coefficient under microgravity ($10^{-4}\,g$).

3. Statistical properties under microgravity

Initially, the systems are prepared for NESS after 30,000 collisions per particle. In NESS, all physical properties are sampled by every 1/10 interval of a vibrating cycle of the plate until $T = 5000$ (i.e. the sample number is 50,000).

3.1 Probability distribution functions

The probability distribution functions (PDFs) of both restitution coefficient and particle velocity under microgravity ($\ll g$) are investigated. Under the gravity on the ground g, we confirmed that there are no difference of PDFs of restitution coefficient and particle velocity between CRC and VDRC.

In figure 3, PDFs of restitution coefficient obtained from simulations by using both CRC ($r_c = 0.92$) and VDRC ($r(v)$, $\varepsilon = 0.92$) at $\Gamma = 10.0$ in microgravity $10^{-4}\,g$ is shown. In case of VDRC, the PDF becomes broad and the mean value moves near the elastic one $r(v) \sim 1$.

Figure 4 shows PDFs of particle velocities by both CRC and VDRC in microgravity $10^{-4}\,g$. In case of CRC, PDF take a maximum at a negative velocity and the asymmetry of the distribution can be found. This is because the particle velocity from the bottom is relatively large. On the other hand, in microgravity $10^{-4}\,g$, PDF becomes Gaussian-like around $v \sim 0$ than that of CRC. This is because the negative feedback mechanism occurs. Namely, when a particle has large velocity, the energy dissipation of collision becomes large, and vise versa.

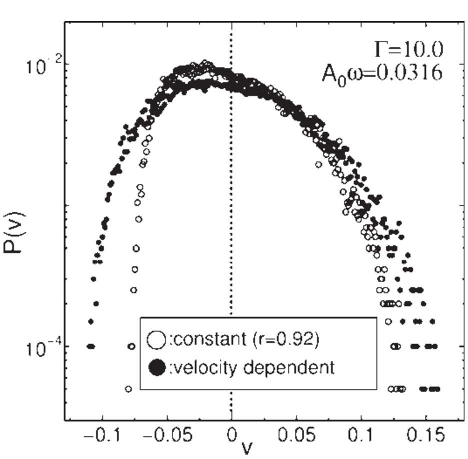

Figure 4. PDF of particle velocity under microgravity ($10^{-4}\,g$).

3.2 Macroscopic properties

To characterize NESS in our model under microgravity quantitatively, we focus on the macroscopic properties (e.g. centre of mass $\bar{Z}$, dilatation λ and dissipation time τ_D), which are studied well by Luding *et al.* [4]. The dilatation λ, which is defined by $\lambda = \langle z_N^* - z_1^* \rangle / A_0\Gamma$, is the ratio of the ensemble average for the expansion of particles between top and bottom $(z_N^* - z_1^*)$ to the squared maximum velocity of a vibrated-plate divided by a fixed gravity $g((A_0\omega)^2/g = A_0\Gamma)$, where $z_i^* = z_i - (i-1)d - (1/2)d$. The dilatation λ takes 0, when particles in the column move collectively as one cluster with contact. Moreover, λ can roughly be regarded as the ratio of the potential energy of the top particle and the kinetic energy of bottom particle on the vibrated plate. The dissipation time $\tau_D = \langle E \rangle / \langle P \rangle$ is the ratio of the averaged total energy of the system $\langle E \rangle$ to the input (or the equivalent output) power $\langle P \rangle$ in NESS. Previous systematic work by using CRC [4] showed that those macroscopic properties can be scaled by functions $F(N, r_c)$ and $A_0\omega$. They showed that the dilatation λ converges to a constant value and the nondimensional dissipation time $\tau_D{\cdot}f$ is proportional to Γ in the fluidized state. Therefore, at a fixed (N, r_c), macroscopic properties take a master (universal) curve in terms of Γ for any (ω, g).

In figures 5 and 6, we show λ and $\tau_D{\cdot}f$ as a function of Γ for $g, 10^{-2}\,g, 10^{-3}\,g, 10^{-4}\,g$ at fixed $(N, \varepsilon, A_0) = (10, 0.92, d)$ by using VDRC. It is found that the behaviour of macroscopic properties is quite different from that of CRC. In microgravity, since PDF of VDRC becomes more elastic, the particles are raised up to high positions and the dissipation of energy per each collision becomes low. At a result, λ and $\tau_D{\cdot}f$ take large values

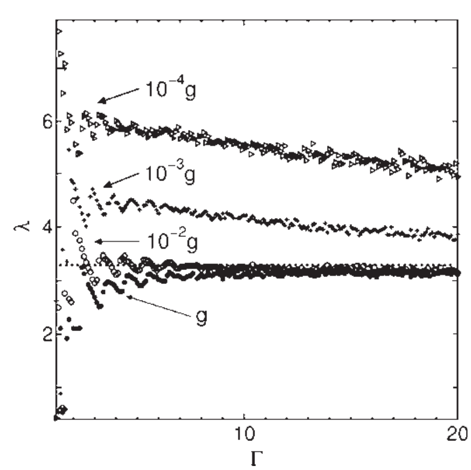

Figure 5. The dilatation λ as a function of Γ by using VDRC under microgravity.

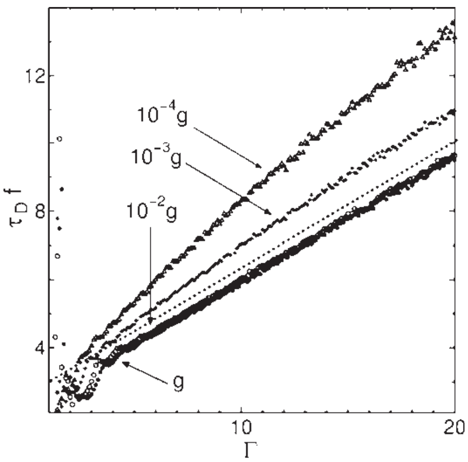

Figure 6. The nondimensional dissipation time $\tau_D{\cdot}f$ as a function of Γ by using VDRC under microgravity.

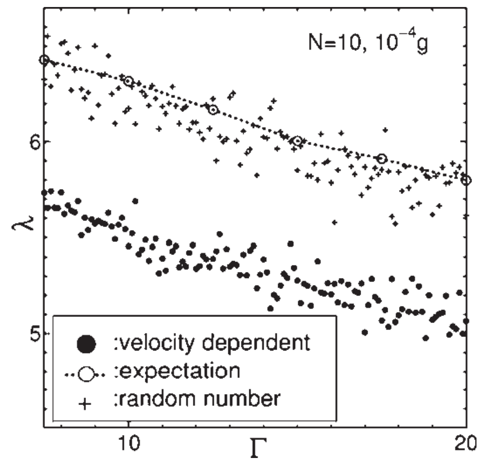

Figure 7. The comparison of three dilatations by direct simulation λ_{sim}, expectation λ_{exp}, and random number λ_{rand}.

in microgravity. Since the vibration response of macroscopic properties increases nonlinearly, they cannot be scaled by simple functions $F(N, r_c)$ and $A_0\omega$ in the high Γ. These results indicate that the simple scaling law of granular system proposed by Luding *et al.* [4] cannot be adopted to present model with VDRC under microgravity.

3.3 Prediction of macroscopic properties based on PDFs

Since the macroscopic properties of the previous simulations take different values for different CRC, it must be confirmed whether the macroscopic properties with VDRC are determined as a linear combination of the results of various CRC. In other word, it is not obvious whether we can predict the macroscopic properties with VDRC based on PDF of VDRC. To confirm this, we compared the dilatations λ obtained from the simulation by using three types of restitution coefficient,

i) λ_{sim} is obtained from the direct simulation with VDRC,
ii) λ_{exp} is estimated by a linear combination of the dilatation $\lambda_c(r)$ of CRC and PDF of VDRC $P_{vd}(r)$, which is described by,

$$\lambda_{\mathrm{exp}} = \int_0^1 \lambda_c(r) \cdot P_{\mathrm{vd}}(r)\mathrm{d}r \qquad (2)$$

iii) λ_{rand} is obtained from the direct simulation with random number restitution coefficient, which is generated based on the same PDF of VDRC.

In figure 7, three dilatations (λ_{sim}, λ_{exp} and λ_{rand}) as a function of Γ under microgravity ($10^{-4}\,g$) are shown. The values of dilatation λ_{sim} (filled-circle) are obviously different from both λ_{rand} (cross) and λ_{exp} (open circle). We obtained same results for other macroscopic properties (i.e. centre of mass $\overline{Z}$ and dissipation time τ_D). Therefore, it is impossible to predict the macroscopic properties based on the PDF of VDRC. On the other hand, λ_{rand} takes almost same values as λ_{exp}. However, in case of $N = 30$, we found that above two dilatations take different values. These results show that the macroscopic properties cannot be predicted even if we have known the PDF of VDRC. In microgravity, the macroscopic properties depend significantly on the microscopic collision rule, which cannot be reproduced by using *artificial* CRC and the restitution coefficient generated by random number. Therefore, we cannot understand the results with VDRC as the extension of previous works. It is significantly important to introduce the realistic model in terms of the restitution coefficient.

3.4 Auto-correlation function and power spectrum

Since it is necessary to investigate the origin of the difference between three macroscopic properties, we focus on the sequence of VDRC in the direct simulation. We calculate auto-correlation function (ACF) for the sequential order of the magnitude of VDRC and its power spectrum. Of course, in case of *artificial* CRC and the restitution coefficient generated by random number, there are no correlations in ACF. Figure 8 shows the power spectrum calculated by the ACF of the direct simulation with VDRC. We found that there are long periodical

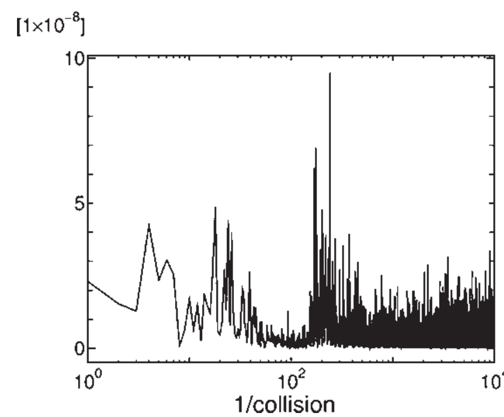

Figure 8. The power spectrum for the sequential order of the magnitude of restitution coefficient.

correlations of collision in the sequence of the magnitude of VDRC under microgravity even in the fluidized state.

4. Concluding remarks

In this paper, we investigate an influence of the macroscopic properties under the competition of two time scale between the (low) impact VDRC of particles and complex many particle dynamics under microgravity. We especially focus on the vibration response of the granular gas and the results obtained from direct simulations with VDRC. In microgravity, the PDFs of VDRC become broad and elastic. The PDF of particle velocities become Gaussian-like because of the negative feedback mechanism. We found that the macroscopic properties obtained from the direct simulation with VDRC under microgravity cannot be explained by a linear combination of the results obtained from the individual simulations with the *artificial* CRC. These results are essentially new, which indicate that the previous works of granular gas with CRC [4] are useless under microgravity. We also calculated the macroscopic properties by using $(1 - r) \propto v^{3/4}$ type of VDRC introduced by Bizon *et al.* [2] (figure 2) and obtained quantitatively different results. These facts indicate that the statistical properties of granular gas under microgravity strongly depend on the microscopic collision rules at the low impact velocity. It is important that our results should be confirmed by the experiments. Our numerical results will provide one of motivation to perform an extensive experiment in microgravity. The time scale of present work corresponds to the experiment in the space shuttle or the International Space Station (ISS).

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